Sample for the first test in Operations Research

1. Solve the given system of linear equations using Gauss-Jordan elimination.

$$\begin{cases} x_1 - x_2 + x_3 - x_4 = -1 \\ x_1 + x_2 - x_3 - x_4 = 3 \\ 2x_1 + x_2 + x_4 = 5 \\ -x_1 + x_3 = -4 \end{cases}$$

2. The Whitt Window Company is a company with only three employees, which makes two different kinds of hand-crafted windows: a wood-framed and an aluminum-framed window. They earn \$60 profit for each wood-framed window and \$30 profit for each aluminum-framed window. Doug makes the wood frames, and can make 6 per day. Linda makes the aluminum frames, and can make 4 per day. Bob forms and cuts the glass, and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass and each aluminum-framed window uses 8 square feet of glass.
    The company wishes to determine how many windows of each type to produce per day to maximize total profit.
    Use the graphical model to solve this problem.

3. Work through the simplex method step by step to solve the following problem.

$$x_1, x_2, x_3 \geq 0$$

$$\begin{aligned} x_1 + x_2 + x_3 &\geq 7 \\ 2x_1 - x_2 + x_3 &\geq 7 \\ x_1 + x_3 &\geq 4 \\ x_2 + 2x_3 &\geq 2 \end{aligned}$$

$$Z(x_1, x_2, x_3) = 10x_1 + 12x_2 + 14x_3 \rightarrow \text{MIN}$$

Answers

1. $x_1 = 2, x_2 = 0, x_3 = -2, x_4 = 1$

2. x_1 – number of wood-framed windows, x_2 – number of aluminum-framed windows
$x_1, x_2 \geq 0$
$x_1 \leq 6$
$x_2 \leq 4$
$6x_1 + 8x_2 \leq 48$
$z(x_1, x_2) = 60x_1 + 30x_2 \rightarrow Max$
Optimal solution: $x_1 = 6$, $x_2 = 1.5$ (int$eger$: 1) , $z_{max} = 405$ (*if* int$eger$:390)

3. $x_1 = 5$, $x_2 = 2$, $x_3 = 0$, $z_{min} = 74$