SOME REMARKS ON MULTILINEAR EXPONENTIAL SUMS WITH AN APPLICATION

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ABSTRACT. A sum-product equation is considered in prime fields. We bound a multilinear exponential sum with an additional requirement for some sets.

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1. Introduction

Let $\mathbb{F}_p$ be the prime field with multiplicative subgroup denoted by $\mathbb{F}_p^*$. A well-known estimation for the double exponential sums is the following upper bound

$$\left| \sum_{x \in X, y \in Y} e(xy) \right| < \sqrt{p|X||Y|},$$

(0.1)

noted by Vinogradov. One of an interesting application of (0.1) is due to Sárközy [6]. He proved that for $A, B, C, D \subseteq \mathbb{F}_p$, the equation

$$a + b = cd, \ (a, b, c, d) \in A \times B \times C \times D$$

has a solution, provided

$$|A||B||C||D| > p^3.$$  (1.1)

A new proof avoids exponential sums was found by Cilleruelo (see [4]).

In [6] the author derived some corollary of this equation; for instance he investigated the Schur type equation

$$a + b = x^k,$$  (1.2)

$a \in A, \ b \in B, x \in \mathbb{F}_p$ i.e. a sumset intersects a subgroup of $\mathbb{F}_p$:

$$A + B \cap H \neq \emptyset; \ H < \mathbb{F}_p.$$  (1.3)

Let us remark here that a deep theorem of Bourgain (see [1]) yields that for every $k$ there exists an $\varepsilon = \varepsilon(k) > 0$ such that (1.3) is solvable when $|A||B| > p^{2-\varepsilon}$ ($\varepsilon$ is ineffective).

In [6] it is also noted that (1.1) is best possible apart the constant factor.

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A more general question would be the investigation of the equations type
\[ a + b = F_p(c, d). \]
Let us mention that it is close to the problem of complete expander polynomials. A polynomial \( f(x_1, \ldots, x_r) \) is said to be complete expander, if there exist \( \delta > 0, \varepsilon = \varepsilon(\delta) > 0 \) such that \(|f(A_1, \ldots, A_r)| \geq \min\{p, |A_r|\}^{1+\varepsilon}\}, \) provided \(|A_1| \leq \cdots \leq |A_r|, |A_r| \gg p^\delta. \)

Indeed it is not too hard to see that the solvability of the equation
\[ a + b = F_p(c, d), |A||B||C||D| > p^\gamma; \quad \gamma > 0 \]
implies that \( f(x, y, z, w) = x + y + F(z, w) \) is a complete expander when \(|A||B||C||D| > p^\gamma\), (namely \( f(A, B, C, D) \) covers \( \mathbb{F}_p \)).

We merely mention that a result of Bourgain also comes from (1.1); he investigated the following question: what is the minimum of the cardinality of \( A \) which ensures that \( 3A^2 = A^2 + A^2 + A^2 = \mathbb{F}_p \) (see [8], [9] and [10]). He concluded that \(|A| > p^{3/4}\) is sufficient.

In [5] we investigate this problem for functions \( F_p(x, y) = x^{1+u}y + x^{2-u}h(y) \) for any \( p \), where we fix \( u \in \{0, 1\} \) and any non constant polynomial \( h(y) \in \mathbb{Z}[y] \), furthermore for \( F_p(x, y) = x^{1+u}y + x^{2-u}g_p \) for any \( p \) where \( g_p \) generates \( \mathbb{F}_p^* \) and \( u \in \{0, 1\} \) is fixed. We proved that if \( F_p \) is one of the two families of functions defined above, then there exist real numbers \( 0 < \delta, \delta' < 1 \) such that for any \( p \) and for any sets \( A, B, C, D \subseteq \mathbb{F}_p \) fulfilling the conditions
\[ |C| > p^{1/2 - \delta}, \quad |D| > p^{1/2 - \delta} |A||B| > p^{2 - \delta'}, \]
there exist \( a \in A, b \in B, c \in C, d \in D \) solving the equation \( a + b = F_p(c, d) \).

In [7] Shparlinski proved that restricting the region of possible values for \(|A|, |B|, |C|, |D|\) one can relax the condition (1.1). He proved that for any fixed \( \varepsilon > 0 \), there exists \( \delta > 0 \) such that if
\[ |A| > p^{1/2 + \varepsilon}, \quad |B| > p^\varepsilon, \quad |C||D| > p^{2 - \delta}, \]
then the equation \( a + b = cd \) can be solved, roughly speaking; restricting the possible region of the sets the power 3 in (1.2) can be decrease to \( \approx 5/2 \). (His proof works actually in arbitrary finite fields.)

In section 2 we investigate any other possibilities to restrict the cardinalities of the sets. We will investigate the 'opposite' of Shparlinski's case; namely when \(|A||B| > p^{2-\alpha}\), \( \alpha \) runs \( 0 < \alpha < 3/16 \) and we decrease the cardinalities of \( C \) and \( D \) under some conditions for the sets (in this case we decrease (1.2) to \( \approx 8/3 \)).

A strong generalization of (0.1) is proved recently by Bourgain. He proved in [1] the following result:
Theorem 1.1. There is a constant $C > 1$ such that for every $0 < \delta < 1$, and $r \in \mathbb{N}$, $r > C/\delta$, if $A_1, A_2, \ldots, A_r \subseteq \mathbb{F}_p$, $|A_i| > p^\delta$ for $1 \leq i \leq r$, $p$ is a prime which is large enough, then

$$\left| \sum_{x_1 \in A_1, \ldots, x_r \in A_r} e(x_1 x_2 \cdots x_r) \right| < p^{-\delta'} |A_1||A_2| \cdots |A_r|,$$

where $\delta' > C^{-r}$.

This important theorem is related to extractors with $\delta$–entropy (see e.g. [3] and [5]).

In Theorem 1.1 when $r$ approaches to infinity then $C^{-r}$ tends to 0 and we can conclude that $\delta' > 0$.

In section 3 we will show that some restriction for three sets of the collection of $A_1, A_2, \ldots, A_r$ we obtain a $\delta'$ depending only on the ratio of the cardinalities of these given three sets.

At this estimation will play a crucial role to give an upper bound for multiplicative energy defined by $E_x(X, Y) = \sum_z r_{X,Y}^2(z)$, where $r_{X,Y}(z) := \{|(x, y) \in X \times Y : z = x \cdot y\}|$.

2. Sárközy’s type sum-product equations

In this section we will consider the following case; let $A, B \subseteq \mathbb{F}_p$ and let $H < \mathbb{F}_p^*$. We ask the solvability of the equation

$$a + b = h; \ (a, b, h) \in A \times B \times H.$$

Restricting the cardinality of $H$ to some region we improve the result of Sárközy:

Theorem 2.1. Let $A, B \subseteq \mathbb{F}_p$, $H < \mathbb{F}_p$. Write $|A||B| = p^{2-2\alpha}; |H| = p^\beta$. Then the equation

$$a + b = h; \ (a, b, h) \in A \times B \times H$$

is solvable, provided

$$\beta > \frac{8\alpha + 1}{3}.$$

Essentially in the same way we can prove a more general result. Assume that $C, D \subseteq \mathbb{F}_p^*$, and assume that the cardinality of the generating subgroups of $C$ and $D$ are close to $|C|$ and $|D|$ respectively. We have

Theorem 2.2. Assume that $C, D \subseteq \mathbb{F}_p^*$, $A, B \subseteq \mathbb{F}_p$. Let $|A||B| = p^{2-2\alpha}; |C| = p^\delta$, $|D| = p^\gamma$, $\langle C \rangle = G_1$, $\langle D \rangle = G_2$, $|G_1| = p^\delta$, $|G_2| = p^\gamma$, $\max\{\delta, \gamma\} < 3/4$. Then the equation

$$a + b = cg \ (a, b, c, g) \in A \times B \times G_1 \times G_2,$$
is solvable, provided
\[ \frac{5}{16} (\beta + \gamma) > \alpha + \frac{1 + \delta + \theta}{8}. \]

**Corollary 1.** Let \( A, B \subseteq \mathbb{F}_p \), \( H < \mathbb{F}_p \). Write \( |H| = p^3 \). Then the equation
\[ a + b = h; \ (a, b, h) \in A \times B \times H \]
is solvable, provided
\[ |A||B||H|^2 > p^{9 + 5\beta}. \]

Note when \( 0 < \beta < \frac{3}{5} \), then it improves the Sárközy’s result.

**Proof of Theorem 2.1**

For the proof we need some lemmas. Recall that \( E_+^s \), the additive energy is defined by
\[ E_+^s(X) = |\{(x_1, \ldots, x_s, x'_1, \ldots, x'_s) \in X^{2s} : x_1 + \cdots + x_s = x'_1 + \cdots + x'_s\}|. \]

**Lemma 2.3.** Let \( C, D \subseteq \mathbb{F}_p \), and let \( S(r) = \sum_{c \in C, g \in D} e(r(c \cdot g)), \ r \in \mathbb{F}_p^* \). Then
\[ |S(r)| \leq |C|^{1/2}|D|^{1/2}(pE_2^+(C)E_2^+(D))^{1/8}. \]

**Remark:** 1. One can prove by induction the more general estimation which sounds as follows: for every \( m, n \in \mathbb{N} \)
\[ |S(r)| \ll_k |C|^{1-1/2^{n+1}}|D|^{1-1/2^m}(pE_{2n+1}^+(C)E_{2m}^+(D))^{1/2^{n+m+1}}. \]

For \( n = 0; \ m = 1 \) it is Lemma 7.1 in [3]. For convenience of the readers we present here the short proof. Note that in the case when \( |C||D| < p \), this estimation is sharper for \( |S(r)| \) than (0.1) and this estimation can be improved if we have an extra information for the additive energy for the sets \( C \), and \( D \) (and some cases the above mentioned generalization also can be applied). Indeed using the fact that for every set \( X \) the bound \( E_+^+(X) < |X|^3 \) holds we obtain
\[ |C|^{1/2}|D|^{1/2}(pE_2^+(C)E_2^+(D))^{1/8} < |C|^{1/2}|D|^{1/2}p^{1/2}. \]

**Proof.** By the triangle inequality and the Cauchy-Schwarz inequality we obtain
\[ |S(r)|^2 \leq |C| \sum_{c \in C} \sum_{g, g' \in D} e(r(c \cdot (g - g'))). \]
Changing the order of the summation and again by the Cauchy-Schwarz
\[ |S(r)|^4 \leq |C|^2|D|^2 \sum_{g, g' \in D} \sum_{c, c' \in C} e(r((c - c') \cdot (g - g'))). \]
Finally using the Vinogradov estimation for the last sum, we obtain the claim of the lemma.

□
A nice application of Stepanov-method ([8] Ch. 9) one can find the following estimation:

**Lemma 2.4.** Let \( G < \mathbb{F}_p^*, |G| \ll p^{3/4}, Y \subseteq G \), then

\[
E_2^+(Y) \ll |G||Y|^{3/2}.
\]

We note that for the proof of Theorem 2.1 we will use the case \( Y = G \); i.e. we use \( E_2^+(Y) \ll |G|^{5/2} \). For the proof of Theorem 2.2 is necessary the sharper form.

Finally we get

**Lemma 2.5.** Assume that for some \( M > 0 \), \( \max_{r \neq 0} |S(r)| \leq M \). If

\[
\sqrt{|A||B||C||D|} > pM,
\]

then the equation \( a + b = cd \ (a, b, c, d) \in A \times B \times C \times D \), is solvable.

The proof of the lemma is simple writing the indicated exponential sum of equation \( a + b = cd \) (see [6]).

Now we are going to give a bound for \( M \).

Firstly we will do it under the condition of Theorem 2.2 and after for the simplicity we end the proof under the condition of Theorem 2.1. Assume that \( C, D \subseteq \mathbb{F}_p^* \) and let the generating subgroup of \( C \) and \( D \), \( \langle C \rangle = G_1, \langle D \rangle = G_2 \) respectively.

By Lemma 2.1 and 2.2 we conclude that

\[
|S(r)| \leq |C|^{1/2}|D|^{1/2}(pE_4^+(C)E_4^+(D))^{1/8} \ll \left\|
\sqrt{|A||B||C||D|} \right\|_{11/16} |G_1|^{1/8} |G_2|^{1/8}.
\]

(2.1)

By Lemma 2.3 we obtain that the equation \( a + b = cd \ (a, b, c, d) \in A \times B \times C \times D \), is solvable, provided

\[
|A|^{1/2}|B|^{1/2}|C|^{5/16}|D|^{5/16} \gg p^{9/8} |G_1|^{1/8} |G_2|^{1/8}.
\]

(2.2)

Writing \( |A||B| = p^{2-2\alpha}, |C| = p^\beta, |D| = p^\gamma, |G_1| = p^\delta, |G_2| = p^\theta \) (2.2) is equivalent to

\[
1 - \alpha + \frac{5}{16} (\beta + \gamma) > \frac{9 + \delta + \theta}{8},
\]

which gives Theorem 2.2. When \( |A||B| = p^{2-2\alpha}, |H| = p^\beta \), it gives the constraint

\[
\beta > \frac{8\alpha + 1}{3}.
\]

and we obtain Theorem 2.1.
3. Multilinear exponential sum with restricted sets

In this section we prove that under some restriction for three sets of the sets $A_1, A_2, A_3, \ldots A_n$ we obtain some explicit bound for a multilinear exponential sum.

**Theorem 3.1.** Let $\varepsilon > 0, p > p(\varepsilon), A_1, A_2, A_3, \ldots A_n \subseteq \mathbb{F}_p, n \geq 3$. Assume that for $i = 2, 3$ $|A_i| \geq c_i \sqrt{p} > 0$,

$$|A_i - A_i| \leq 8c_i^2 |A_i|,$$

and

$$0 < \alpha \leq \frac{\ln(|A_1|/(|A_2||A_3|)^{13/8+\varepsilon})}{2 \ln p} + 5/8.$$

Then

$$|S| := \left| \sum_{x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n} e(x_1 \cdots x_n) \right| < p^{-\alpha} \cdot \prod_{i=1}^{n} |A_i|.$$

**Corollary 2.** Let $|A_2|, |A_3| \asymp \sqrt{p}, |A_1| > p^{3/8}$ and assume (3.1) holds. Then

$$|S| := \left| \sum_{x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n} e(x_1 \cdots x_n) \right| < p^{-\alpha} \cdot \prod_{i=1}^{n} |A_i|,$$

where $0 < \alpha < \frac{\ln|A_1|}{2 \ln p} - \frac{3}{16}$.

**Proof of Theorem 3.1**

We use the notation $\hat{f}(y) = \sum_x f(x) \cdot e(xy)$. Write the sum

$$S = \sum_{x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n} e(x_1 \cdots x_n) = \sum_{x_2 \in A_2, x_3 \in A_3, \ldots, x_n \in A_n} \sum_{x_1 \in A} e(x_1 (x_2 \cdots x_n)) =$$

$$= \sum_{x_2 \in A_2, x_3 \in A_3, \ldots, x_n \in A_n} \hat{A}_1(x_2 \cdots x_n) = \sum_{x_2, x_3, \ldots, x_n \in \mathbb{F}_p} A_2(x_2) \cdots A_n(x_n) \hat{A}_1(x_2 \cdots x_n),$$

where $A_i(x_i)$ is the indicator of the set $A_i$. Write $z = x_2 \cdots x_n$, then we have

$$S = \sum_{z \in \mathbb{F}_p} r(z) \hat{A}_1(z),$$

where

$$r(z)_{A_1, \ldots, A_n} = r(z) =$$

$$= |\{(x_2, \ldots, x_n) : x_2 \in A_2, x_3 \in A_3, \ldots, x_n \in A_n; z = x_2 \cdots x_n\}|.$$
Thus by the Cauchy-Schwarz and the Parseval

\[ |S| \leq \sqrt{\sum_{z \in \mathbb{F}_p} r^2(z)} \cdot \sqrt{\sum_{z \in \mathbb{F}_p} |\tilde{A}(z)|^2} = \sqrt{E_x(A_2, A_3, \ldots, A_n)} \cdot \sqrt{p} |A_1|, \]

where \( E_x(A_2, A_3, \ldots, A_n) \) is the multiplicative energy of the sets \( A_2, A_3, \ldots, A_n \). The \( n \) terms multiplicative energy is defined by

\[ E_x(X_1, \ldots, X_n) = \]

\[ = |\{(x_1, \ldots, x_n, x'_1, \ldots, x'_n) \in (X_1 \times \cdots \times X_n)^2 : x_1 \cdots x_n = x'_1 \cdots x'_n\}|. \]

**Lemma 3.2.** Let \( X_1, X_2, \ldots, X_n \subseteq \mathbb{F}_p \) and denote the multiplicative energy of them by \( E_x(X_1, \ldots, X_n) \).

We have

\[ E_x(X_1, \ldots, X_n) \leq |X_1|^2 E_x(X_2, \ldots, X_n). \]

**Proof.** By the definition of the multiplicative energy

\[ E_x(X_1, \ldots, X_n) = \]

\[ = |\{(x_1, \ldots, x_n, x'_1, \ldots, x'_n) \in (X_1 \times \cdots \times X_n)^2 : x_1 \cdots x_n = x'_1 \cdots x'_n\}| \]

hence

\[ E_x(X_1, \ldots, X_n) = \]

\[ = | \bigcup_{x_1, x'_1 \in X_1} \{(x_2, \ldots, x_n, x'_2, \ldots, x'_n) \in (X_2 \times \cdots \times X_n)^2 : x_1 \cdots x_n = x'_1 \cdots x'_n\}|. \]

Fix a pair \( x_1, x'_1 \in X_1 \). Now the set

\[ \{(x_2, \ldots, x_n, x'_2, \ldots, x'_n) \in (X_2 \times \cdots \times X_n)^2 : x_1 \cdots x_n = x'_1 \cdots x'_n\} \]

can be written as

\[ \{(x_2, \ldots, x_n, x'_2, \ldots, x'_n) \in (X_2 \times \cdots \times X_n)^2 : x_2 \cdots x_n = (x_1^{-1} x'_1) x_2 \cdots x'_n\} \]

Hence

\[ |\{(x_2, \ldots, x_n, x'_2, \ldots, x'_n) \in (X_2 \times \cdots \times X_n)^2 : x_2 \cdots x_n = (x_1^{-1} x'_1) x_2 \cdots x'_n\}| = \]

\[ = \sum_{z \in \mathbb{F}_p} r'(z) r'(x_1^{-1} x'_1 z) \]

where \( r'(u) = |\{(x_2, \ldots, x_n) : x_2 \in A_2, x_3 \in A_3, \ldots, x_n \in A_n; u = x_2 \cdots x_n\}|. \)

Finally by using Cauchy-Schwarz we obtain

\[ E_x(X_1, \ldots, X_n) \leq \]

\[ \leq \sum_{x_1, x'_1 \in X_1} |\{(x_2, \ldots, x_n, x'_2, \ldots, x'_n) \in (X_2 \times \cdots \times X_n)^2 : x_1 \cdots x_n = x'_1 \cdots x'_n\}| \leq \]

\[ \leq \sum_{x_1, x'_1 \in X_1} \sum_{z \in \mathbb{F}_p} r'(z) r'(x_1^{-1} x'_1 z) \leq |X_1|^2 \sqrt{\sum_{z \in \mathbb{F}_p} r'^2(z)} \sqrt{\sum_{z \in \mathbb{F}_p} r'^2((x_1^{-1} x'_1) z)} = \]
\[ |X_1|^2 \sum_{z \in \mathbb{F}_p} r^2(z) = |X_1|^2 E_x(X_2, \ldots, X_n), \]

using the fact if \( z \) runs on \( \mathbb{F}_p \) then \((x_1^{-1} x_1') z\) so does. \( \square \)

Iterating the result of Lemma 3.2 we can estimate
\[ |S| \leq \sqrt{E_x(A_2, A_3) \cdot \sqrt{p|A_1|}}. \]

**Lemma 3.3.** Let \( B, C \subseteq \mathbb{F}_p^* \). Then
\[ E_x(B, C) \leq \sqrt{E_x(B, B) E_x(C, C)}. \]

This lemma is Corollary 2.10 in [8] proved for additive energy. The proof for multiplicative energy is the same and short, so we prove for seek of completeness.

**Proof.** Let \( q_{U,V}(n) := |\{(u, v) \in U \times V : n = u/v\}|. \) Clearly
\[ \sum_z q_{B,C}^2(z) = \sum_z q_{C,B}^2(z) = \sum_z r_{B,B}(z) r_{C,C}(z), \]
since
\[ \sum_z q_{B,C}^2(z) = \sum_z q_{C,B}^2(z) = \sum_z r_{B,C}(z) r_{C,B}(z) = \]
\[ = |\{(b, b', c, c') \in B \times B \times C \times C : b/c = b'/c'\}| = \]
\[ = |\{(b, b', c, c') \in B \times B \times C \times C : c/b = c'/b'\}| = \]
\[ = |\{(b, b', c, c') \in B \times B \times C \times C : cb' = bc'\}|. \]
By the Cauchy-Schwarz inequality we obtain the result. \( \square \)

Now by Lemma 3.3 we have with \( B = A_2, C = A_3, \)
\[ |S| \leq \prod_{i=4}^n |A_i|(E_x(A_2, A_2))^{1/4} \cdot (E_x(A_3, A_3))^{1/4} \cdot \sqrt{p|A_1|}. \quad (3.3) \]

**Lemma 3.4.** Let \( U \subseteq \mathbb{F}_p \). Assume that \( |U - U| \leq 8\frac{|U|^3}{p} \)
\[ E_x(U, U) \leq 2^{9} \frac{|U|^{29/4}}{p^{9/4}} \ln |U|. \quad (3.4) \]
Proof. For the proof of (3.4) we use Theorem 1.1 in [3]:

Let $U \subseteq \mathbb{F}_p$. We have

$$E_x(U, U) \leq 2\sqrt{2} \sqrt[4]{|U - U| + \frac{8|U|^3}{p} |U|^{5/4}|U - U| \sqrt{|2U - 2U| \ln |U|}}. \tag{3.5}$$

Now by the Plünnecke-Ruzsa inequality and the bound $|U - U| \leq \frac{8|U|^3}{p}$ we have

$$|2U - 2U| \leq \frac{|U - U|^4}{|U|^3} \leq \frac{2^{12}|U|^9}{p^4}.$$ 

and using the bound $|U - U| \leq \frac{8|U|^3}{p}$ again in (3.5) an easy calculation gives the upper estimation for $E_x(U, U)$. \hfill \square

Now we turn to the estimation of $|S|$. By (3.1) we can use (3.5) for the sets $A_2$ and $A_3$. We obtain

$$|S| \leq \prod_{i=4}^n |A_i|(E_x(A_2, A_2))^{1/4} \cdot (E_x(A_3, A_3))^{1/4} \cdot \sqrt{p|A_1|} \leq$$

$$\leq 4 \prod_{i=4}^n |A_i| \sqrt{p|A_1|} \left(\frac{|A_2||A_3|}{p^{9/8}}\right)^{29/16} \ln |A_2| \ln |A_3| =$$

$$= 4 \prod_{i=4}^n |A_i||A_1||A_2||A_3| p^{-\alpha} p^{\alpha - 5/8} |A_1|^{1/2} \left(\frac{|A_2||A_3|}{p^{9/8}}\right)^{13/16} \ln |A_2| \ln |A_3|.$$ 

Thus from

$$p^{\alpha - 5/8} |A_1|^{-1/2} \left(\frac{|A_2||A_3|}{p^{9/8}}\right)^{13/16} \ln |A_2| \ln |A_3| \leq 1, \tag{3.6}$$

we obtain

$$|S| \leq 4 \prod_{i=4}^n |A_i||A_1||A_2||A_3| p^{-\alpha} = 4 \prod_{i=1}^n |A_i| p^{-\alpha}. \tag{3.6}$$

(3.6) holds if

$$|A_1| \geq p^{2\alpha - 5/4} \left(\frac{|A_2||A_3|}{p^{9/8}}\right)^{13/8} \ln^2 |A_2| \ln^2 |A_3|. \tag{3.7}$$

When $|A_2|, |A_3| \asymp \sqrt{p}$ we obtain

$$|A_1| \geq p^{2\alpha + 3/8 + \epsilon}. \tag{3.8}$$

From (3.7) and (3.8) an easy calculation gives (3.2) and the Corollary.

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